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The light quark masses

Paul B. Mackenzie[a]

[a] Fermilab, P.O. Box 500, Batavia, IL 60510 USA

I discuss the improvement program for obtaining the physical, short-distance light quark masses from experiment using lattice methods.

1. INTRODUCTION

The masses of the light quarks are fundamental parameters of the standard model which are results of QCD spectrum calculations. They are in some ways more interesting than determinations of the strong coupling constant, since they are known from conventional phenomenology only to within a factor of around three, whereas the strong coupling constant was known in advance from deep inelastic scattering and other methods to 10% or so.

2. IMPROVED COUPLING AND MASS

An improvement program for these coupling constants may be defined which is parallel to, but distinct from, the improvement program for the strong coupling constant on the lattice.[1] Several stages of improved couplings may be considered:

1. A one loop perturbative relation to the bare quark mass [2–4]:

$$m_{\overline{MS}}(\pi/a) = m_0(1 + c\alpha) \quad (1)$$

2. A mean field improved relation with the bare quark mass [1,5], analogous to the mean field improved strong coupling constant used in Ref. [6]:

$$m_{\overline{MS}}(\pi/a) = \frac{m_0}{\sqrt[4]{< U_P >_{MC}}}\,(1 + c'\alpha) \quad (2)$$

3. A nonperturbative extraction of the renormalized quark mass analogous to the nonperturbatively extracted strong coupling constants of Refs. [1] and [7]. Such methods have not yet been developed for the quark masses as they have for α_s.

It is possible to absorb the corrections for the mass operator into the scale of the operator, as they conventionally are for α_s (when Λ_{lat} is related to $\Lambda_{\overline{MS}}$). The bulk of the corrections for α_s are not renormalization group logarithms, but rather power law mean field "tadpoles". Nevertheless, absorbing them into the scale is a convenient and not misleading bookkeeping device, since both types of corrections sum up as geometric series. This is not the case for corrections to the mass operator. Different sources of corrections imply different estimates for higher order terms, and so the different types of corrections need to be treated separately.

The expression for the renormalized mass after mean field improvement may be written

$$\begin{aligned} & m_{\overline{MS}}(\mu) \\ =\; & m_0\left[1 + g^2\gamma_0\left(\ln C_m - \ln(a\mu)\right)\right] && (3) \\ =\; & \tilde{m}\left[1 + g^2\left(\gamma_0\left(\ln C_m - \ln(a\mu)\right) - \frac{1}{12}\right)\right] && (4) \\ \equiv\; & \tilde{m}\left[1 + g^2\left(\gamma_0\left(\ln\tilde{C}_m - \ln(a\mu)\right)\right)\right] && (5) \end{aligned}$$

where

$$\tilde{m} \equiv \frac{m_0}{U_0} = \frac{m_0}{1 - \frac{1}{12}g^2} \quad (6)$$

in perturbation theory, and

$$\tilde{m} \equiv \frac{m_0}{\sqrt[4]{< U_P >_{MC}}} \quad (7)$$

nonperturbatively, if the plaquette is used to define the mean link. The numerical values of these coefficients are given in the table. The expression for Wilson fermions is well behaved after mean field improvement, while a large correction (about 40% if a renormalized g^2 is used) remains

	C_m	$\tilde{C}_m$
Wilson fermions	8.66	1.67
Staggered fermions	689.5	132.9

for staggered fermions. The appropriate scale for the mass operator may be estimated using techniques analogous to those for calculating the appropriate scale for the strong coupling constant described in Ref. [1]. While the calculation for estimating the appropriate scale for the mass operator has not yet been done, it is implausible that it will yield the scale of $132.9/a$ which is required to explain the one loop correction. The bulk of the staggered fermion correction is a mystery at present, which undermines confidence in the use of perturbation theory in this case.

3. EFFECTS OF QUENCHING

The effects of the quenched approximation are quite different for the three types of fundamental parameters of QCD. For the strong coupling constant, the incorrect β function of the quenched approximation produces a mismatch between scale of low energy physics and the higher energy scales at which the coupling is extracted. If α_s is determined from heavy quark bound states, simple Coulomb gluon exchange at relatively high energies (around a GeV in the case of the Υ system) is expected to dominate the physics to a much greater extent than is true for light hadrons. The effects of quenching may therefore be estimated in advance of first principles inclusion of light quark loops. The case of the b and c quark masses is greatly simplified by the fact that "running mass" stops running for momentum transfers smaller than the pole mass of the quark. Therefore, the effective mass governing the physics is approximately the pole mass, whether quenched or unquenched.[8]

The case of the light quark masses is hardest of all. Perturbation theory leads us to expect a definite effect from short distance physics. (α_s is a bit too small in the quenched approximation, so the quarks don't pick up quite as much mass renormalization as they should.) However, there is no reason not to expect a contribution from the lower energy scales of the quarks and gluons inside pions, which we have no way of estimating in a reasonable way. It is instructive nevertheless to calculate the short distance effects of the quenched approximation. These effects dominate as the lattice spacing is taken to zero (if the quark mass is obtained from lattice spacing scale physics), although for medium lattice spacings there is no guarantee that they do.

The ratio of the one-loop running of the mass between a low and a high momentum scale in the quenched approximation and in the full theory may be written

$$\frac{\left.\frac{m(low)}{m(high)}\right|_{\text{quenched}}}{\left.\frac{m(low)}{m(high)}\right|_{\text{unquenched}}} \approx \frac{\left(\frac{\alpha_0(low)}{\alpha_0(high)}\right)^{\gamma_0/(2\beta_0^{(0)})}}{\left(\frac{\alpha_3(low)}{\alpha_3(high)}\right)^{\gamma_0/(2\beta_0^{(3)})}}. \tag{8}$$

To estimate the part of this expression arising from short distance effects, we assume that the parameters of quenched and unquenched lattice calculations have been set to get the light hadron physics as right as possible. We thus assume that the effective coupling constants and running masses are approximately the same in the quenched and unquenched theories at the low energy scales. (These are murkier assumptions for light hadrons than they are for heavy quark bound states.) We make the further simplifying and not unreasonable assumption that $\alpha \approx 1$ at these scales. We then obtain, to leading logarithmic accuracy,

$$\begin{aligned} \frac{m(high)|_{\text{qu.}}}{m(high)|_{\text{unqu.}}} &\approx \alpha(high)^{\frac{\gamma_0}{2}(1/\beta_0^{(0)}-1/\beta_0^{(3)})} && (9) \\ &\approx 1.15 \text{ to } 1.20, && (10) \end{aligned}$$

for $\alpha(high) \approx 1/6$ to $1/8$. The nonperturbative contributions to this result cannot be easily estimated. Therefore, for the light quark masses, and as opposed to the case of α_s, unquenched calculations must be considered from the beginning.

4. DISCUSSION OF NUMERICAL RESULTS

Numerical results for the light quark masses have been summarized by A. Ukawa.[9] The results for staggered fermions are approximately independent of the lattice spacing, as they should be, while Wilson fermion results vary and slowly approach the staggered results as the lattice spacing is reduced. A possible but untested explanation is that $O(a)$ errors in the Wilson action are responsible. It is unfortunate that the numerical results for staggered fermions appear better than those for Wilson fermions, since the required perturbation is very well-behaved for Wilson fermions and more dubious for staggered fermions. Two-flavor unquenched results for staggered fermions are roughly 20% to 30% below the quenched results, which is not unreasonable in light of Eqn. 10. Boldly (or recklessly) taking these results for staggered fermions seriously, we extrapolate to the three flavor result for staggered fermions. Taking $m_0 = 1.40$ MeV $\pm$ 9% (statistics) for the two flavor bare mass, and $m_0 =$ 1.95 MeV in the quenched approximation[10], we obtain for the three flavor result

$$m_0 = 1.40\sqrt{\frac{1.40}{1.95}} = 1.19 \text{ MeV} \pm 15\%(stat). \quad (11)$$

For the renormalized mass we then obtain

$$\begin{aligned} & m_{\overline{MS}}(1 \text{ GeV}) && (12)\\ = \;& 1.19 \text{ MeV} \cdot 1.20 \cdot 1/0.87 \cdot 1.41 && (13)\\ = \;& 2.3 \text{ MeV}, && (14) \end{aligned}$$

where the three factors in Eqn. 13 arise respectively from the continuum running to 1 GeV, the tadpole contribution to the mass renormalization, and the unexplained part of the one loop result for staggered fermions. For the strange quark mass, this implies a value of $m_s(1\text{GeV}) = m_l \cdot 27 \approx 65$ MeV. These estimates are below and outside the generous bounds for light quark masses given by conventional phenomenology.

Several topics need more work before uncertainty estimates can be ventured. They include the calculation of the appropriate scale for the mass operator, the extrapolation in the number of flavors to $n_f = 3$, and the reliability of perturbation in the case of staggered fermions.

5. CONCLUSIONS

The determination of m_l is thus harder than the determination of α_s in several ways.

- There is no way of estimating the effects of the quenched approximation in advance of first principles calculations.
- Methods for extracting the short distance mass parameter nonperturbatively (analogous to those proposed for the strong coupling constant in Refs. [1,7]) have not been developed.
- Perturbation theory is worse behaved for the fermions (staggered) for which the numerical data seem well behaved.

On the other hand, the payoff will ultimately be larger: The strong coupling constant was known in advance from deep inelastic scattering to 10% or so; the light quark masses are known to only around a factor of three.

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